16.3 MOLE FRACTION AND MOLALITY

Caution: Be sure distinguish carefully between molarity and molality. There is only one- letter difference in the spelling of the words, and the difference in the symbols is the difference between M and m, but:

Molarity is the number of the moles of solute per *liter of solution.*

Molality is the number of moles of solute per *kilogram of solvent.*

16.70 Calculate the mole fraction of water in a mixture consisting of 9.0 g water, 120 g acetic acid, and 115 g ethyl alcohol.

- The molecular weights of water acetic acid, and ethyl alcohol are 18, 60, and 46 g/mol, respectively. The mixture thus contains 0.50 mol water, 2.0 mol acetic acid, and 2.5 mol ethyl alcohol, for a total of 5 mol:

$$x(H_2O) = \frac{0.50}{5.0} = 0.10$$

In other words, one out of each 10 molecules in the solution is a water molecule.

16.71 What is the mole fraction of H_2 in a gaseous mixture containing 1.0 g H_2, 8.0 g O_2, and 16 g CH_4 ?

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$$(1.0\text{ g } H_2)\left(\frac{1\text{ mol } H_2}{2.0\text{ g } H_2}\right) = 0.50\text{ mol } H_2 \qquad (8.0\text{ g } O_2)\left(\frac{1\text{ mol } O_2}{32.0\text{ g } O_2}\right) = 0.25\text{ mol } O_2$$

$$(16\text{ g } CH_4)\left(\frac{1\text{ mol } CH_4}{16.0\text{ g } CH_4}\right) = 1.0\text{ mol } CH_4$$

$$x(H_2) = \frac{0.50\text{ mol } H_2}{1.75\text{ mol total}} = 0.29$$

16.72 A solution contains 116 g acetone (CH_3COCH_3), 138 g ethyl alcohol (C_2H_5OH), and 126 g water. Determine the mole fraction of each.

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$$(116\text{ g } C_3H_6O)\left(\frac{1\text{ mol } C_3H_6O}{58.0\text{ g } C_3H_6O}\right) = 2.00\text{ mol } C_3H_6O$$

$$(138\text{ g } C_2H_5OH)\left(\frac{1\text{ mol } C_2H_5OH}{46.0\text{ g } C_2H_5OH}\right) = 3.00\text{ mol } C_2H_5OH \qquad (126\text{ g } H_2O)\left(\frac{1\text{ mol } H_2O}{18.0\text{ g } H_2O}\right) = 7.00\text{ mol } H_2O$$

$$\text{Total} = 2.00 + 3.00 + 7.00 = 12.00\text{ mol}$$

$$x(C_3H_6O) = \frac{2.00}{12.00} = 0.167 \qquad x(C_2H_5OH) = \frac{3.00}{12.00} = 0.250 \qquad x(H_2O) = \frac{7.00}{12.00} = 0.583$$

16.73 A solution contains 18.0 g glucose (MW = 180), 24 g acetic acid (MW = 60), and 81 g water (MW =18). What is the mole fraction of acetic acid in the solution?

$$\frac{18.0\text{ g glucose}}{180\text{ g/mol}} = 0.100\text{ mol glucose} \qquad \frac{24\text{ g acetic acid}}{60\text{ g/mol}} = 0.40\text{ mol acetic acid}$$

$$\frac{81\text{ g water}}{18\text{ g/mol}} = 4.5\text{ mol water} \qquad x(\text{acetic acid}) = \frac{0.40}{4.5 + 0.4 + 0.1} = 0.080$$

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16.74 Determine the mole fraction of both substances in a solution containing 36.0 g water and 46.0 g glycerin, $C_3H_5(OH)_3$.

- The molecular weight of $C_3H_5(OH)_3$ is 92.0; of H_2O, 18.0.

$$n(\text{glycerin}) = \frac{46.0\text{ g}}{92.0\text{ g/mol}} = 0.500\text{ mol} \qquad n(\text{water}) = \frac{36.0\text{ g}}{18.0\text{ g/mol}} = 2.00\text{ mol}$$

$$\text{Total number of moles} = 0.50 + 2.00 = 2.50\text{ mol}$$

$$x(\text{glycerin}) = \text{mole fraction of glycerin} = \frac{n(\text{glycerin})}{\text{total number of moles}} = \frac{0.50}{2.50} = 0.20$$

$$x(\text{water}) = \text{mole fraction of water} = \frac{n(\text{water})}{\text{total number of moles}} = \frac{2.00}{2.50} = 0.80$$

16.75 The density of a 2.0 M solution of acetic acid (MW = 60) in water is 1.02 g/mL. Calculate the mole fraction of acetic acid.

- Per liter of solution: $(2.0\text{ mol acid})(60.0\text{ g/mol}) = 120\text{ g acid}$

$$(1.000\text{ L})(1.02\text{ kg/L}) = 1.02\text{ kg solution} = 1020\text{ g solution}$$

$$(1020\text{ g solution}) - (120\text{ g acid}) = 900\text{ g water}$$

$$(900\text{ g }H_2O)\left(\frac{1\text{ mol }H_2O}{18.0\text{ g }H_2O}\right) = 50\text{ mol }H_2O \qquad x(\text{acid}) = \frac{2.0\text{ mol}}{(50\text{ mol}) + (2.0\text{ mol})} = 0.038$$

16.76 a solution contains 10.0 g acetic acid, CH_3COOH, in 125 g water. What is the concentration of the solution expressed as (a) mole fraction of CH_3COOH and H_2O (b) molality?

- **(a)** $(10.0\text{ g }HC_2H_3O_2)\left(\dfrac{1\text{ mol }HC_2H_3O_2}{60.0\text{ g }HC_2H_3O_2}\right) = 0.167\text{ mol }HC_2H_3O_2$

  $(125\text{ g }H_2O)\left(\dfrac{1\text{ mol }H_2O}{18.0\text{ g }H_2O}\right) = 6.94\text{ mol }H_2O$

  $x(\text{acid}) = \dfrac{0.167}{0.167 + 6.94} = 0.0235 \qquad x(\text{water}) = 1.0000 - 0.0235 = 0.9765$

  (b) $\dfrac{0.167\text{ mol }HC_2H_3O_2}{0.125\text{ kg }H_2O} = 1.34\text{ m}$

16.77 Calculate the molalities and the mole fractions of acetic acid in two solutions prepared by dissolving 120 g acetic acid (a) in 100 g water (b) in 100 g ethyl alcohol.

- **(a)** In water:

  $$\text{Molality} = \frac{2.00\text{ mol acetic acid}}{0.10\text{ kg water}} = 20\text{ m} \qquad x(\text{acetic acid}) = \frac{2.00\text{ mol}}{(2.00 + 5.55)\text{ mol}} = 0.265$$

(b) In ethyl alcohol:

$$\text{Molality} = \frac{2.00\text{ mol acetic acid}}{0.10\text{ kg ethyl alcohol}} = 20\text{ m} \qquad x(\text{acetic acid}) = \frac{2.00\text{ mol}}{(2.00 + 2.17)\text{ mol}} = 0.480$$

16.78 The density of a 2.03 M solution of acetic acid in water is 1.017 g/mL. Calculate the molality of the solution.

- One liter of the solution, containing 2.03 mole solute, has a mass of 1017 g. thus the solution contains 2.03 mol x 60.0 g/mol = 122 g acetic acid and also 1017 – 122 = 895 g water. The molality is

$$\frac{2.03\text{ mol solute}}{0.895\text{ kg water}} = 2.27\text{ m}$$

16.79 What mass of ammonium chloride is dissolved in 100 g water in each of the following solutions? (a) 1.10 m NH_4Cl solution (b) A solution which is 75 % water by mass (c) A solution with a mole fraction of 0.15 NH_4Cl.

- $$(a)\quad (100\ \mathrm{g\ H_2O})\left(\frac{1.10\ \mathrm{mol}}{1000\ \mathrm{g\ H_2O}}\right)\left(\frac{53.5\ \mathrm{g}}{\mathrm{mol}}\right) = 5.88\ \mathrm{g} \qquad (b)\quad (100\ \mathrm{g\ H_2O})\left(\frac{25.0\ \mathrm{g\ NH_4Cl}}{75.0\ \mathrm{g\ H_2O}}\right) = 33.3\ \mathrm{g}$$
  $$(c)\quad (100\ \mathrm{g\ H_2O})\left(\frac{1\ \mathrm{mol\ H_2O}}{18.0\ \mathrm{g\ H_2O}}\right) = 5.55\ \mathrm{mol\ H_2O} \qquad (5.55\ \mathrm{mol\ H_2O})\left(\frac{0.15\ \mathrm{mol\ NH_4Cl}}{0.85\ \mathrm{mol\ H_2O}}\right)\left(\frac{53.5\ \mathrm{g}}{\mathrm{mol\ NH_4Cl}}\right) = 52.5\ \mathrm{g}$$

16.80 What is the molality of a solution which contains 20.0 g cane sugar, $C_{12}H_{22}O_{11}$, dissolved in 125 g water?

- The molecular weight of $C_{12}H_{22}O_{11}$ is 342.
  $$m = \text{molality} = \frac{n(\text{solute})}{\text{mass of solvent in kg}} = \frac{(20.0\ \mathrm{g})/(342\ \mathrm{g/mol})}{0.125\ \mathrm{kg}} = 0.468\ \mathrm{mol/kg}$$

16.81 The molality of a solution of ethyl alcohol, C_2H_5OH, in water is 1.54 mol/kg. How many g alcohol is dissolved in 2.50 kg water?

- The molecular weight of C_2H_5OH is 46.1 g/mol. Since the molality is 1.54. 1 kg water dissolves 1.54 mol alcohol. Then 2.50 kg water dissolves (2.50)(1.54) = 3.85 mol alcohol, and
  Mass of alcohol = (3.85)(46.1 g/mol) = 177 g alcohol

16.82 Calculate the (a) molar concentration and (b) molality of a sulfuric acid solution of density 1.198 g/cm^3, containing 27.0 % H_2SO_4 by weight.

- (a) Each cm^3 of acid solution has a mass of 1.198 g and contains (0.270)(1.198) = 0.324 g H_2SO_4. Since the molecular weight of H_2SO_4 is 98.1,

$$M = \frac{n(\mathrm{H_2SO_4})}{\text{volume of solution in L}} = \frac{(0.324\ \mathrm{g})/(98.1\ \mathrm{g/mol})}{(1\ \mathrm{cm^3})(10^{-3}\ \mathrm{L/cm^3})} = 3.30\ \mathrm{mol/L} = 3.30\ \mathrm{M}$$

(*b*) From (*a*), there is 324 g, or 3.30 mol, of solute per liter of solution. The amount of water in 1 L of solution is 1198 g − 324 g = 874 g H_2O. Hence

$$m = \frac{n(\text{solute})}{\text{mass of solvent in kg}} = \frac{3.30\ \text{mol}\ H_2SO_4}{0.874\ \text{kg}\ H_2O} = 3.78\ \text{mol/kg} = 3.78\ \text{m}$$

16.83 Calculate the molality of a solution containing (a) 0.65 mol glucose, $C_6H_{12}O_6$, in 250 g water (b) 45 g glucose in 1.00 kg water (c) 18 g glucose in 200 g water.

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$$(a)\quad \frac{0.65\ \text{mol}}{0.250\ \text{kg}} = 2.6\ \text{m} \qquad (b)\quad \left(\frac{45\ \text{g glucose}}{\text{kg water}}\right)\left(\frac{1\ \text{mol glucose}}{180\ \text{g glucose}}\right) = 0.25\ \text{m}$$

$$(c)\quad \left(\frac{18\ \text{g glucose}}{200\ \text{g water}}\right)\left(\frac{1\ \text{mol glucose}}{180\ \text{g glucose}}\right)\left(\frac{10^3\ \text{g}}{\text{kg}}\right) = 0.50\ \text{m}$$

16.84 How many g $CaCl_2$ should be added to 300 mL water to make up a 2.46 m solution?

- Assuming that the water has a density of 1.00 g/mL,

$$(300\ \text{mL water})\left(\frac{1.00\ \text{g water}}{\text{mL water}}\right)\left(\frac{1\ \text{kg}}{10^3\ \text{g}}\right)\left(\frac{2.46\ \text{mol}}{\text{kg}}\right)\left(\frac{111\ \text{g}\ CaCl_2}{\text{mol}}\right) = 81.9\ \text{g}\ CaCl_2$$

16.85 A solution contains 57.5 mL ethyl alcohol (C_2H_5OH) and 600 mL benzene (C_6H_6). How many g alcohol are in 1000 g benzene? What is the molality of the solution? Density of C_2H_5OH is 0.800 g/mL; of C_6H_6, 0.900 g/mL.

$$(57.5\ \text{mL}\ C_2H_5OH)\left(\frac{0.800\ \text{g}}{\text{mL}}\right) = 46.0\ \text{g}\ C_2H_5OH \qquad (600\ \text{mL}\ C_6H_6)\left(\frac{0.900\ \text{g}}{\text{mL}}\right)\left(\frac{1\ \text{kg}}{1000\ \text{g}}\right) = 0.540\ \text{kg}\ C_6H_6$$

$$\frac{46.0\ \text{g}\ C_2H_5OH}{0.540\ \text{kg}\ C_6H_6} = \frac{85.2\ \text{g}\ C_2H_5OH}{\text{kg}\ C_6H_6} \qquad \left(\frac{85.2\ \text{g}\ C_2H_5OH}{\text{kg}}\right)\left(\frac{1\ \text{mol}\ C_2H_5OH}{46.0\ \text{g}\ C_2H_5OH}\right) = 1.85\ \text{mol/kg} = 1.85\ \text{m}$$

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16.86 What is the mole fraction of the solute in a 1.00 m aqueous solution?

There is 1.00 mol solute in 1.00 kg H_2O.

$$(1.00\ \text{kg})\left(\frac{10^3\ \text{g}}{\text{kg}}\right)\left(\frac{1\ \text{mol}\ H_2O}{18.0\ \text{g}\ H_2O}\right) = 55.6\ \text{mol}\ H_2O \qquad x = \frac{1.00}{1.00 + 55.6} = 0.0177$$

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16.87 an aqueous solution labeled 35.0% $HClO_4$ had a density of 1.251 g/cm^3. What are the molar concentration and molality of the solution?

- Consider 1.000 L of solution: Its mass is 1.251 kg = 1251 g. It contains

$$(1251 \text{ g solution})\left(\frac{35.0 \text{ g } HClO_4}{100.0 \text{ g solution}}\right)\left(\frac{1 \text{ mol } HClO_4}{100.5 \text{ g } HClO_4}\right) = 4.36 \text{ mol } HClO_4$$

$$\frac{4.36 \text{ mol } HClO_4}{1.251 \text{ kg } (0.650)} = 5.36 \text{ m} \qquad 4.36 \text{ mol/L} = 4.36 \text{ M}$$

16.88 a sucrose solution was prepared by dissolving 0.0135 kg $C_{12}H_{22}O_{11}$ in enough water to make exactly 0.1000 L solution, which was then found to have a density of 1.050 kg/L. Compute the molar concentration and molality of the solution.

$$(0.0135 \text{ kg } C_{12}H_{22}O_{11})\left(\frac{1 \text{ mol } C_{12}H_{22}O_{11}}{342 \text{ g } C_{12}H_{22}O_{11}}\right)\left(\frac{1000 \text{ g}}{\text{kg}}\right) = 0.0395 \text{ mol } C_{12}H_{22}O_{11}$$

- $$(0.1000 \text{ L})\left(\frac{1.050 \text{ kg}}{\text{L}}\right) = 0.1050 \text{ kg solution} \qquad (0.1050 \text{ kg}) - (0.0135 \text{ kg}) = 0.0915 \text{ kg } H_2O$$

$$\frac{0.0395 \text{ mol}}{0.100 \text{ L}} = 0.395 \text{ M} \qquad \frac{0.0395 \text{ mol}}{0.0915 \text{ kg}} = 0.432 \text{ m}$$

16.89 Determine the volume of dilute nitric acid (density 1.11 g/mL, 19 % HNO_3 by weight) that can be prepared by diluting with water 50 mL of the

$$(50.0 \text{ mL solution})\left(\frac{1.42 \text{ g solution}}{\text{mL solution}}\right)\left(\frac{69.8 \text{ g } HNO_3}{100 \text{ g solution}}\right) = 49.6 \text{ g } HNO_3$$

$$(49.6 \text{ g } HNO_3)\left(\frac{100 \text{ g solution}}{19.0 \text{ g } HNO_3}\right)\left(\frac{1 \text{ mL}}{1.11 \text{ g}}\right) = 235 \text{ mL} \qquad (49.6 \text{ g } HNO_3)\left(\frac{1 \text{ mol } HNO_3}{63.0 \text{ g } HNO_3}\right) = 0.787 \text{ mol } HNO_3$$

$$\frac{0.787 \text{ mol}}{0.0500 \text{ L}} = 15.7 \text{ M} \qquad \frac{0.787 \text{ mol}}{0.235 \text{ L}} = 3.35 \text{ M} \qquad (50.0 \text{ mL})\left(\frac{1.42 \text{ g}}{\text{mL}}\right) = 71.0 \text{ g solution}$$

$$\text{Mass of } H_2O = 71.0 \text{ g} - 49.6 \text{ g} = 21.4 \text{ g } H_2O \qquad \frac{0.787 \text{ mol}}{0.0214 \text{ kg}} = 36.8 \text{ m}$$

$$(235 \text{ mL})\left(\frac{1.11 \text{ g}}{\text{mL}}\right) = 261 \text{ g solution} \qquad \text{Mass of } H_2O = 261 \text{ g} - 49.6 \text{ g} = 211 \text{ g } H_2O$$

$$\frac{0.787 \text{ mol}}{0.211 \text{ kg}} = 3.73 \text{ m}$$

concentrated acid (density 1.42 g/mL, 69.8% HNO_3 by weight). Calculate the molar concentration and molalities of the concentrated and dilute acids.

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16.90 Calculate the molarity, molality, and mole fraction of ethyl alcohol in a solution of total volume 95 mL prepared by adding 50 mL ethyl alcohol (density = 0.789 g/mL) to 50 mL water (density = 1.00 g/mL). Calculate the molarity of water in alcohol.

- $$(50\ \text{mL})\left(\frac{0.789\ \text{g}}{\text{mL}}\right)\left(\frac{1\ \text{mol}}{46.0\ \text{g}}\right) = 0.86\ \text{mol}\ C_2H_5OH \qquad (50\ \text{mL})\left(\frac{1.0\ \text{g}}{\text{mL}}\right)\left(\frac{1\ \text{mol}}{18\ \text{g}}\right) = 2.8\ \text{mol}\ H_2O$$

$$\text{Molality} = \frac{0.86\ \text{mol}\ C_2H_5OH}{0.050\ \text{kg}\ H_2O} = 17\ \text{m}$$

$$\text{Mole fraction} = \frac{0.86\ \text{mol}}{0.86\ \text{mol} + 2.8\ \text{mol}} = 0.23 \qquad \text{Molarity} = \frac{0.86\ \text{mol}}{0.095\ \text{L}} = 9.1\ \text{M}$$

$$\text{Molality of water} = \frac{2.8\ \text{mol}\ H_2O}{(0.050\ \text{L})(0.789\ \text{kg/L})\ C_2H_5OH} = 71\ \text{m}$$

16.91 For a solute of molecular weight W, show that the molar concentration M and molality m of the solution are related by

$$M\left(\frac{W}{1000}+\frac{1}{m}\right)=d$$

Where d is the solution density in g/mL. (Hint: show that each mL of solution contains MW/1000 g of solute and M/mg of solvent.) Use this relation to check the answer to Problems 16.87 and 16.88.

The number of g solute per mL solution is given by

$$\left(\frac{M\ \text{mol}}{\text{L}}\right)\left(\frac{1\ \text{L}}{10^3\ \text{mL}}\right)\left(\frac{W\ \text{g}}{\text{mol}}\right)=\frac{MW}{1000}\ \text{g solute/mL soln}$$

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The number of g solvent per mL solution is given by

$$\left(\frac{M\ \text{mol solute}}{\text{L}}\right)\left(\frac{\text{kg solvent}}{m\ \text{mol solute}}\right)\left(\frac{\text{L}}{10^3\ \text{mL}}\right)\left(\frac{10^3\ \text{g}}{\text{kg}}\right)=\frac{M}{m}\ \text{g solvent/mL soln}$$

$$\text{Density}=\frac{\text{total g}}{\text{mL}}=\frac{MW}{1000}+\frac{M}{m}=M\left(\frac{W}{1000}+\frac{1}{m}\right)$$

16.92 Show algebraically that the sum of the mole fractions of all components of a solution must equal 1.00.

- In a solution of $A + B + C + \cdots$,

$$x(\text{A})=\frac{a\ \text{mol A}}{(a\ \text{mol A})+(b\ \text{mol B})+(c\ \text{mol C})+\cdots}$$

Similarly for the mole fractions of B and C, etc., all the fractions having the same denominator. The total of all the mole fractions is thus

$$\frac{(a\text{ mol A}) + (b\text{ mol B}) + (c\text{ mol C}) + \cdots}{(a\text{ mol A}) + (b\text{ mol B}) + (c\text{ mol C}) + \cdots} = 1$$

16.93 The density of 10.0 % by mass KCl solution in water is 1.06 g/mL. Calculate the molarity, molality, and mole fraction of KCl in this solution.

- In each 100.0 g of solution are

$$(10.0\text{ g KCl})\left(\frac{1\text{ mol}}{74.5\text{ g}}\right) = 0.134\text{ mol KCl} \qquad \text{and} \qquad (90.0\text{ g }H_2O)\left(\frac{1\text{ mol}}{18.0\text{ g}}\right) = 5.00\text{ mol }H_2O$$

$$(100\text{ g solution})\left(\frac{1\text{ mL}}{1.06\text{ g}}\right) = 94.3\text{ mL} = 0.0943\text{ L}$$

$$\text{Molarity} = \frac{0.134\text{ mol KCl}}{0.0943\text{ L}} = 1.42\text{ M} \qquad \text{Molality} = \frac{0.134\text{ mol KCl}}{0.0900\text{ kg }H_2O} = 1.49\text{ m}$$

$$\text{Mole fraction} = \frac{0.134\text{ mol}}{(0.134\text{ mol}) + (5.00\text{ mol})} = 0.0261$$